Stats - III Theory

MN10 ADE

stats - III

10/10/13

TIME : 2 Hrs MARKS : 60

Note :
1. **All questions are compulsory.**
2. **Bracketed figures to the right indicate marks.**
3. **Graph paper will be provided on request.**

**Q.1 a.** For the following Linear programming problem, state in which quadrant the feasible region will exist. **(1)**

Max $z = 4x + 3y$

Subject to $2x + 3y \leqslant 4$

$3x + y \leqslant 5$

$x \geqslant 0, y \geqslant 0$

**OR**

**a.** Define "Slack Variable" **(1)**

**b. Attempt any two questions out of 3 from the following.**

**i.** Two nutrients $V_1$ and $V_2$ are found in two foods $F_1$ and $F_2$. One unit of $F_1$ contains 5 units of $V_1$ and 30 units of $V_2$. One unit of $F_2$ contains 20 units of $V_1$ and 20 units of $V_2$. Minimum daily requirement of $V_1$ and $V_2$ is 100 and 300 units respectively. **(7)**

Cost per unit of $F_1$ and $F_2$ is ₹. 5 and ₹. 10 respectively. Formulate the L.P.P. & Obtain optimal solution using graphical method.

**ii.** Solve the following linear programming problem using simplex method. Determine whether feasible solution exists. **(7)**

Max $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leqslant 2$

$3x_1 + 4x_2 \geqslant 12$

$x_1, x_2 \geqslant 0$

**iii.** Solve the following Linear programming problem using simplex method. **(7)**

Max $Z = 40x_1 + 50x_2$

Subject to

$2x_1 + 3x_2 \leqslant 3$

$8x_1 + 4x_2 \leqslant 5$

$x1 \geqslant 0 , x_2 \geqslant 0$

Write the dual of the above L.P.P. Hence write the optimal basic feasible solution from the optimal simplex table of the primal.

**Q.2 a. Fill in the blank by selecting the correct alternative.**

The number of non - negative variables in a basic feasible solution to a $m \times n$ transportation problem is ............ **(1)**

**i.** mn **ii.** m + n **iii.** m + n + 1 **iv.** m + n - 1

**a.** One disadvantage of using North - West corner Rule to find initial solution to the transportation problem is that: **(1)**

**i.** it is complicated to use.

**ii.** it does not take into account the cost of transportation

**iii.** it leads to a degenerate initial solution.

**iv.** all of the above.

**Q.2 b. Attempt any two questions out of three from the following.**

**i.** Explain Vogel's Approximation Method of solving a transportation problem. **(7)**

**ii.** Find initial feasible solution to the following transportation problem using matrix minima method. **(7)**

| From To / Factory | Warehouses $W_1$ | $W_2$ | $W_3$ | $W_4$ | Supply |
|---|---|---|---|---|---|
| $F_1$ | 42 | 32 | 50 | 26 | 11 |
| $F_2$ | 34 | 36 | 28 | 46 | 13 |
| $F_3$ | 64 | 54 | 36 | 82 | 19 |
| Demand | 6 | 10 | 12 | 15 | |

**iii.** Given below is a table taken from the solution process for a transportation problem. **(7)**

| Factory | Distribution Centre I | II | III | IV | Available (Units) |
|---|---|---|---|---|---|
| A | 10 | 8 / 5000 | 7 | 12 | 5000 |
| B | 12 | 13 | 6 / 4500 | 10 / 1500 | 6000 |
| C | 8 / 7000 | 10 / 500 | 12 | 14 / 1500 | 9000 |
| Demand in units | 7000 | 5500 | 4500 | 3000 | |

Answer the following questions, giving brief answers based on above transportation table:

**i.** Is this solution feasible.

**ii.** Is this solution degenerate?

**iii.** Is this solution optimum? If not, find the optimum solution?

**Q.3 a. Fill in the blank by selecting the correct alternative.**

The purpose of a dummy row or column in an assignment problem is to ............... **(1)**

1. to obtain balance between total activities and total resources
2. prevent a solution from becoming degenerate
3. Provide the means of representing a dummy problem
4. none of the above

**OR**

a. The assignment problem is ...............

1. requires that only one activity be assigned to each resources
2. is a special case of transportation problem
3. can be used to maximize resources
4. all of the above

**Q.3 b. Attempt any two questions out of three from the following.**

i. Explain the Hungarian method to solve an assignment problem. **(7)**

ii. In a textile sales emporium four salesman A, B, C and D are available to four counters W, X, Y and Z. Each salesman can handle any counter. The service (in hour) of each counter when manned by each salesman is given below. **(7)**

| Counter | Salesman | | | |
|---|---|---|---|---|
| | A | B | C | D |
| W | 41 | 72 | 39 | 52 |
| X | 22 | 29 | 49 | 65 |
| Y | 27 | 39 | 60 | 51 |
| Z | 45 | 50 | 48 | 52 |

How should the salesman be allocated appropriate counters so as to minimize the service time? Each salesman must handle only one counter.

iii. Find the sequence that minimizes the total elapsed time required to complete the following tasks. **(7)**

| Tasks | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Time on $I^{st}$ Machine | 3 | 8 | 7 | 4 | 9 | 8 | 7 |
| Time on $II^{nd}$ Machine | 4 | 3 | 2 | 5 | 1 | 4 | 3 |
| Time on $III^{rd}$ Machine | 6 | 7 | 5 | 11 | 5 | 6 | 12 |

**Q.4 a.** Define artificial variable. **(1)**

**OR**

a. Define unbalanced transportation problem. **(1)**

**b. Attempt any two questions out of three from the following.**

i. The simplex table in the process of obtaining the optimal solution is **(7)** given below for the linear programming problem.

Minimize $z = x_1 - 3x_2 + 2x_3$

Subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Simplex table:

| Basic Variable | $C_B$ | $X_B$ | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | 10 | 5/2 | 0 | 2 | 1 | 1/4 | 0 |
| $X_2$ | 3 | 3 | -1/2 | 1 | 0 | 0 | 1/4 | 0 |
| $S_3$ | 0 | 1 | -5/2 | 0 | 8 | 0 | -3/4 | 1 |

Test whether the solution in the above simplex tables is optimum. If not determine the Key row, Key column and Key element.

**ii.** Find the initial basic feasible solution to the following transportation problem using North West corner rule. Where $O_i$ and $D_j$ represents $i^{th}$ origin and $j^{th}$ destination. (i = 1, 2, 3, 4), (j = 1, 2, 3) **(7)**

| | Destination | | | |
|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | Supply |
| From $O_1$ | 2 | 7 | 4 | 5 |
| $O_2$ | 3 | 3 | 1 | 8 |
| $O_3$ | 5 | 4 | 7 | 7 |
| $O_4$ | 1 | 6 | 2 | 14 |
| Demand | 7 | 9 | 18 | |

**iii.** We have five jobs each of which must go through two machines in the order AB. Processing times are given in the table below. **(7)**

| Job No. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Machine A | 10 | 2 | 18 | 6 | 20 |
| Machine B | 4 | 12 | 14 | 16 | 8 |